PCME

A LEVEL COACH, TUTOR

2025 Singapore-Cambridge A Level

H2 Math Paper 2

Suggested Answer Key

PREFACE

Dear students taking the Singapore-Cambridge A Level, the published solution below is personally written by Mr Mitch Peh, who is a former MOE JC lecturer and has taught at both NYJC and SAJC.

Mr Peh is a holder of 5 A Stars at the Cambridge International A level, scoring almost 100% of the total marks for Physics, Chemistry, Mathematics, Economics and Further Mathematics.

If you are considering 1 to 1 tuition to improve your A level subjects results, you can sign up with Mr Peh.

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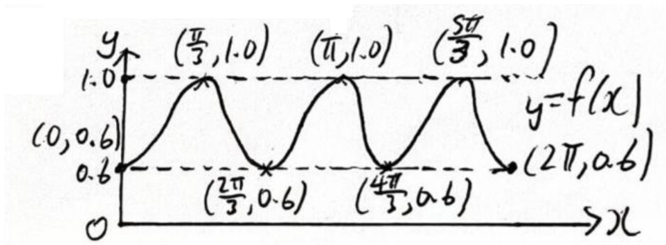
Section A: Pure Mathematics [40 marks]

1 (a)	$8c+11t+5b=114.....(1)$ $5c+14t+7b=112.....(2)$ $9c+9t+4b=110.....(3)$ Using GC to solve (1), (2) and (3) simultaneously, we have: $c = 7$, $t = 3$, $b = 5$	[3]
(b)	Since each player scored a different number of points and Sam came second, Sam must have scored 113 points. Also, we require $z > y$ as there are more boats than trains. Hence, z has to be at least 7 as y is at least 6. Let the number of cars, trains and boats that Sam has be x, y and z respectively. $7x+3y+5z=113$ Suppose $x = 6$ and $y = 6$, $7(6)+3(6)+5z\le 113$ $5z\le 53$ $z\le 10.6$ Since $7\le z\le 10.6$ and z is an integer, the possible values of z are 7, 8, 9 and 10. x and y need to be integers and at least 6. When $z = 10$, $x=\frac{63-3y}{7}$ From GC, $y = 7$, $x = 6$ When $z = 9$, $x=\frac{68-3y}{7}$ From GC, there is no possible solution When $z = 8$, $x=\frac{73-3y}{7}$ From GC, $y = 8$, $x = 7$ However, we need $z > y$ so this solution cannot be accepted. When $z = 7$, $x=\frac{78-3y}{7}$ From GC, there is no possible solution Alternatively, When $z = 10$, $y=\frac{63-7x}{3}$ From GC, $x = 6$, $y = 7$ When $z = 9$, $y=\frac{68-7x}{3}$ From GC, there is no possible solution When $z = 8$, $y=\frac{73-7x}{3}$ From GC, $y = 8$, $x = 7$ However, we need $z > y$ so this solution cannot be accepted. When $z = 7$, $y=\frac{78-7x}{3}$ From GC, $x = 6$, $y = 12$ However, we need $z > y$ so this solution cannot be accepted Hence, the number of cars, trains and boats that Sam has are be 6, 7 and 10 respectively. Comments Students can key in the 4 expressions into y= of the GC simultaneously and observe with the table in the GC.	[3]

2 (a)	Let the position vector of R be $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. $3\overrightarrow{QR} = 4\overrightarrow{PQ}$ $3\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix}\right] = 4\left[\begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}\right]$ $\begin{pmatrix} 3x-3 \\ 3y-18 \\ 3z-18 \end{pmatrix} = 4\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$ Hence, $3x-3=12 \Rightarrow x=5$ $3y-18=12 \Rightarrow y=10$ $3z-18=24 \Rightarrow z=14$ $\therefore \overrightarrow{OR} = \begin{pmatrix} 5 \\ 10 \\ 14 \end{pmatrix}$	[3]
(b)	$\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\overrightarrow{SQ} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix}$ Since $\overrightarrow{SQ}$ is perpendicular to $\overrightarrow{PQ}$, $\overrightarrow{PQ} \cdot \overrightarrow{SQ} = 0$ $\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix} = 0$ $6+24+36-6c=0$ $6c=66$ $c=11$	[2]

(c)	Let $\angle PSQ = \theta$ $\overrightarrow{PS} = \begin{pmatrix} -1 \\ -2 \\ 11 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix}$ $\theta = \cos^{-1} \dfrac{\left\lvert \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\rvert}{\left\lvert \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\rvert}$ $\theta = \cos^{-1} \dfrac{18}{\sqrt{147}\sqrt{6}}$ $\theta = 52.692^o$ $\theta = 52.7^o$ <u>Comments</u> Note that $\overrightarrow{OS} = \begin{pmatrix} -1 \\ -2 \\ 11 \end{pmatrix}$ while $\overrightarrow{SQ} = \begin{pmatrix} 2 \\ 8 \\ -5 \end{pmatrix}$. Some students may use the incorrect vector for $\overrightarrow{OS}$ here.	[3]

3 (a)	$\dfrac{dy}{dx} = y^2 \sin 3x$ $\int \dfrac{1}{y^2} dy = \int \sin 3x dx$ $-\dfrac{1}{y} = \dfrac{-\cos 3x}{3} + C$ When $x = \pi$, $y = 1$ $-1 = \dfrac{-\cos 3\pi}{3} + C$ $-1 = \dfrac{1}{3} + C$ $C = -\frac{4}{3}$ $-\dfrac{1}{y} = \dfrac{-\cos 3x - 4}{3}$ $y = \dfrac{3}{4 + \cos 3x}$ Hence, A = 3, B = 4	[4]

(b) (i)	 From graph, the values of *c* are 0.6 and 1.0	[3]
(ii)	The equations of axes of symmetry: $x = \frac{k\pi}{3}, k \in \mathbb{Z}$ This is because: $\cos x$ is symmetrical about $k\pi, k \in \mathbb{Z}$ $\cos 3x$ is symmetrical about $\frac{k\pi}{3}, k \in \mathbb{Z}$ The same will apply for $y = f(x) = \frac{3}{4+\cos 3x}$ <u>Comments</u> Note that the domain $0 \le x \le 2\pi$ only applies for the sketching of the graph. In contrast, the domain for $y = f(x)$ is $x \in \mathbb{R}$.	[2]
(iii)	$f(x+\pi) = \frac{3}{4+\cos(3x+3\pi)}$ $= \frac{3}{4+\cos 3x \cos 3\pi - \sin 3x \sin 3\pi}$ $= \frac{3}{4-\cos 3x - 0}$ $= \frac{3}{4-\cos 3x}$ <u>Comments</u> Need to apply the compound angle formula given in MF27 here.	[1]
(c)	$\frac{dy}{dx} = y^2 \sin 3x....(1)$ $\frac{d^2y}{dx^2} = 2y\left(\frac{dy}{dx}\right)\sin 3x + 3y^2 \cos 3x....(2)$ Subst (1) in (2), we have: $\frac{d^2y}{dx^2} = 2y^3 \sin^2 3x + 3y^2 \cos 3x$ $\frac{d^2y}{dx^2} = (3\cos 3x)y^2 + (2\sin^2 3x)y^3$ Hence, P(*x*) = 3cos3*x*, Q(*x*) = 2 $\sin^2 3x$	[3]

	Comments Students need to remember about applying chain rule when performing differentiation.	

4 (a)	$V=\frac{1}{3}\pi h^2(45-h)$ The bowl is filled when h = 15 cm. $V=\frac{1}{3}\pi(15)^2(30)$ $=2250\pi cm^3$ Time taken $=\frac{2250\pi}{10\pi}=225s$ Comments When the bowl is filled, the volume will be the same as the volume of a hemisphere as $V=\frac{1}{3}\pi(15)^2(30)=\frac{2}{3}\pi(15)^3$	[2]
(b)	$V=\frac{1}{3}\pi h^2(45-h)=15\pi h^2-\frac{1}{3}\pi h^3$ $\frac{dV}{dh}=30\pi h-\pi h^2$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $\frac{dh}{dt}=\frac{10\pi}{30\pi h-\pi h^2}$ When h = 12cm, $\frac{dh}{dt}=\frac{10}{30(12)-(12)^2}=\frac{10}{216}=\frac{5}{108}$ Hence, the rate at which the depth of water is increasing when the depth of water is 12cm is $\frac{5}{108}$ cms^{-1}.	[4]
(c)	$\frac{dV}{dt}=10\pi t$ $\int 1dV=10\pi\int tdt$ $V=5\pi t^2+C$ When t = 0, V = 0. Hence, C = 0 When $V=972\pi cm^3$, $972\pi=5\pi t^2$ $t^2=194.4$ $t=13.942=13.9s(3s.f.)$	[3]

(d)	$V=\frac{1}{3}\pi h^2(45-h)$ When $V=972\pi cm^3$, $972\pi=\frac{1}{3}\pi h^2(45-h)$ $2916=45h^2-h^3$ $h^3-45h^2+2916=0$ From GC, h = 43.455 (rej), -7.4558 (rej) or 9 cm as $0\le h\le 15$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $\frac{dh}{dt}=\frac{10\pi t}{30\pi h-\pi h^2}$ When h = 9, t = 13.942, $\frac{dh}{dt}=\frac{(10\pi)(13.942)}{30\pi(9)-\pi(9)^2}$ $\frac{dh}{dt}=0.73767$ $\frac{dh}{dt}=0.738cms^{-1}$	[4]

Section B: Probability and Statistics [60 marks]

5 (a)	Method 1a: P&C Direct Method P(R,B) + P(R,Y) + P(B, Y) $=\frac{{}^3C_1\times{}^5C_1\times{}^7C_0}{{}^{15}C_2}+\frac{{}^3C_1\times{}^5C_0\times{}^7C_1}{{}^{15}C_2}+\frac{{}^3C_0\times{}^5C_1\times{}^7C_1}{{}^{15}C_2}$ $=\frac{1}{7}+\frac{1}{5}+\frac{1}{3}$ $=\frac{71}{105}$ Method 1b: Probability Direct Method P(R,B) + P(R,Y) + P(B, Y) $=\left(\frac{3}{15}\right)\left(\frac{5}{14}\right)(2!)+\left(\frac{3}{15}\right)\left(\frac{7}{14}\right)(2!)+\left(\frac{5}{15}\right)\left(\frac{7}{14}\right)(2!)$ $=\frac{1}{7}+\frac{1}{5}+\frac{1}{3}$ $=\frac{71}{105}$	[3]

Method 2a: P&C Complement Method

1 – P(R, R) – P(B, B) – P(Y, Y)

$$=1-\frac{{}^{3}C_2\times{}^{5}C_0\times{}^{7}C_0}{{}^{15}C_2}-\frac{{}^{3}C_0\times{}^{5}C_2\times{}^{7}C_0}{{}^{15}C_2}-\frac{{}^{3}C_0\times{}^{5}C_0\times{}^{7}C_2}{{}^{15}C_2}$$

$$=1-\frac{1}{35}-\frac{2}{21}-\frac{1}{5}$$

$$=\frac{71}{105}$$

Method 2b: P&C Complement Method

$$=1-\left(\frac{3}{15}\right)\left(\frac{2}{14}\right)-\left(\frac{5}{15}\right)\left(\frac{4}{14}\right)-\left(\frac{7}{15}\right)\left(\frac{6}{14}\right)$$

$$=1-\frac{1}{35}-\frac{2}{21}-\frac{1}{5}$$

$$=\frac{71}{105}$$

<u>Comments</u>

For method 1b and 2b, students need to remember the selection is without replacement. Hence, after the first selection, there is only 14 counters left.

(b) [2]

There are 3 red counters and 12 non-red counters

Probability that first red is second counter he takes

$$=\left(\frac{12}{15}\right)\left(\frac{3}{14}\right)$$

$$=\frac{78}{455}$$

$$=\frac{6}{35}$$

Probability that first red is third counter he takes

$$=\left(\frac{12}{15}\right)\left(\frac{11}{14}\right)\left(\frac{3}{13}\right)$$

$$=\left(\frac{6}{35}\right)\left(\frac{11}{13}\right)$$

$$=\frac{66}{455}$$

Probability that first red is fourth counter he takes

$=\left(\frac{12}{15}\right)\left(\frac{11}{14}\right)\left(\frac{10}{13}\right)\left(\frac{3}{12}\right)$

$=\left(\frac{6}{35}\right)\left(\frac{11}{13}\right)\left(\frac{10}{12}\right)$

$=\frac{55}{455}$

$=\frac{11}{91}$

Probability that first red is fifth counter he takes

$=\left(\frac{12}{15}\right)\left(\frac{11}{14}\right)\left(\frac{10}{13}\right)\left(\frac{9}{12}\right)\left(\frac{3}{11}\right)$

$=\left(\frac{6}{35}\right)\left(\frac{11}{13}\right)\left(\frac{10}{12}\right)\left(\frac{9}{11}\right)$

$=\frac{45}{455}$

$=\frac{9}{91}$

Probability that first red is sixth counter he takes

$=\left(\frac{12}{15}\right)\left(\frac{11}{14}\right)\left(\frac{10}{13}\right)\left(\frac{9}{12}\right)\left(\frac{8}{11}\right)\left(\frac{3}{10}\right)$

$=\left(\frac{6}{35}\right)\left(\frac{11}{13}\right)\left(\frac{10}{12}\right)\left(\frac{9}{11}\right)\left(\frac{8}{10}\right)$

$=\frac{36}{455}$

Hence, the value of n = 6.

	Comments To derive the formula to solve this question, it may take even longer time or incorrectly done for some students. Hence, I would recommend the trial and error method instead of the formula method shown below. Probability that first red is nth counter he takes $=\left(\frac{12}{15}\right)\left(\frac{11}{14}\right)\left(\frac{10}{13}\right)\ldots\left(\frac{12-n+2}{15-n+2}\right)\left(\frac{3}{15-n+1}\right)$ $=\frac{\frac{12!}{(12-n+1)!}}{\frac{15!}{(15-n)!}}\times 3$ $=\frac{(15-n)!}{(13-n)!}\times\frac{12!}{15!}\times 3$ $=\frac{3(15-n)(14-n)}{(15)(14)(13)}=\frac{36}{455}$ $(15-n)(14-n)=72$ $210-29n+n^2=72$ $n^2-29n+138=0$ $n=23(rej)$ or $n=6$	

6 (a)	$P(7<X<\mu)=0.45$ $P(X<\mu)-P(X<7)=0.45$ $P(X<7)=P(X<\mu)-0.45$ $P(X<7)=0.5-0.45=0.05$	[1]
(b)	$X\sim N(\mu,1.8^2)$ $P(X<7)=0.05$ $P\left(Z<\frac{7-\mu}{1.8}\right)=0.05$ $\frac{7-\mu}{1.8}=-1.6448$ $7-\mu=-2.96064$ $\mu=9.96064$ $\mu=9.96(3s.f.)$ Comments Key in invNorm(0.05, 0, 1, left) to obtain Z value of -1.6448.	[2]

(c)	Since X and Y are independent, $P((X>7)\cap(Y>7)) = P(X>7)P(Y>7) = 0.38$ $P(Y>7) = \dfrac{0.38}{P(X>7)}$ $P(Y>7) = \dfrac{0.38}{1-0.05}$ $P(Y>7) = 0.40$ Given $Y \sim N\left(\lambda, 2.8^2\right)$, $P(Z > \dfrac{7-\lambda}{2.8}) = 0.40$ $\dfrac{7-\lambda}{2.8} = 0.25335$ $7-\lambda = 0.70938$ $\lambda = 6.29062$ $\lambda = 6.29$ <u>Comments</u> Key in invNorm(0.40, 0, 1, right) to obtain 0.25335.	[3]
(d)	$Y \sim N\left(6.29062, 2.8^2\right)$ $P\left(Y>a\right) = 2P\left(Y>7\right)$ $P\left(Y>a\right) = 0.80$ Using GC, a = 3.9340 a = 3.93 <u>Comments</u> Key in invNorm(0.80, 6.29062, 2.8, right) to obtain a value of 3.9340.	[2]

7 (a)	Number of different arrangements $= \dfrac{8!}{2!2!} = 10080$	[2]
(b)	Number of different arrangements $= {}^3C_2 \times 2! \times {}^4C_1 \times \dfrac{5!}{2!2!} = 720$ <u>Comments</u> A common mistake is to forget accounting for repeated odd digits of ‘3’ and ‘7’. First, we select the 2 even digits to be at the start and the end. Second, we arrange these 2 even digits. Third, there are 4 possible slots to for the last even digit to be positioned at. Fourth, we arrange the 5 odd numbers.	[3]

(c) [2]

Number of different arrangements

$$=\frac{6!}{2!2!}\times 3! = 1080$$

Comments

First, we arrange the 5 blocks of odd digits and 1 block of even digits.
Then, we arrange the 3 even digits within the 1 block.

(d) [1]

P(odd digits are all together | even digits are all together)

$$=\frac{P(\text{odd digits are all together}\cap\text{even digits are all together})}{P(\text{even digits are all together})}$$

$$=\frac{n(\text{odd digits are all together}\cap\text{even digits are all together})}{n(\text{even digits are all together})}$$

$$=\frac{2!\times\frac{5!}{2!2!}\times 3!}{1080}$$

$$=\frac{360}{1080}$$

$$=\frac{1}{3}$$

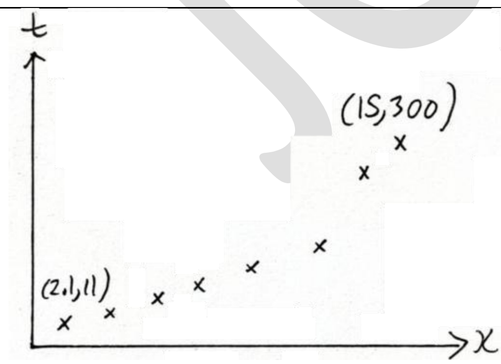
Comments

First we arrange the 2 blocks of odd and even digits.
Second, we arrange the odd digits.
Third, we arrange the even digits.

8(a) [2]

- The product moment correlation coefficient of 0.39 is positive but not close to 1.
- Hence, this suggests a weak positive linear correlation between best performance in high jump and long jump amongst athletes taking part in both events.
- In addition, from the scatter diagram, when the performance in high jump is better, the performance in long jump does not necessarily get better as well.
- Nevertheless, the scatter diagram shows that the data are clustered together at the top right corner of the graph. This suggests that when the athlete is good at high jump, he is good at long jump as well.

(b)(i) [2]



The scatter diagram suggests a curvilinear relationship between x and t.
As x increases, t increases at an increasing rate.

<table>
<tr><td></td><td><u>Comments</u>
• Key in the data of x and t into L1 and L2 columns of the GC table.
• Press Stat Plot on
• Press 9: Zoom Stat</td><td></td></tr>
<tr><td>(ii)
(A)</td><td>From GC,
For $t = ax + b$, $r = 0.95751$
For $t = cx^2 + d$, $r = 0.99046$

Since the product moment correlation coefficient of the model $t = cx^2 + d$ is 0.990 which is closer to one compared to the product moment correlation of 0.958 of the model $t = ax + b$, there is a stronger positive linear correlation between x^2 and t compared to the correlation between x and t. Hence, $t = cx^2 + d$ model is the better fit to the data.
In addition, for the model $t = cx^2 + d$, as x increases, t increases at an increasing rate which fits the scatter diagram better. In contrast, for the model $t = ax + b$, as x increases, t increases at a constant rate instead.

<u>Comments</u>
Key into GC L3 as $L1^2$.
Use either 4:LinReg($ax + b$) or 8: LinReg($a + bx$) under the stat function in GC to obtain product moment correlation coefficient value.</td><td>[2]</td></tr>
<tr><td>(B)</td><td>From GC,
regression equation for the model identified in part (ii)(A)
$t = 1.2496x^2 + 0.87580$
$t = 1.25x^2 + 0.876$

<u>Comments</u>
Similarly, use either 4: LinReg($ax + b$) or 8: LinReg($a + bx$) under the stat function in GC to obtain the equation.</td><td>[2]</td></tr>
<tr><td>(iii)
(A)</td><td>When $x = 11$,
$t = 1.2496(11)^2 + 0.87580$
$t = 152.05$
$t = 152$ minutes</td><td>[1]</td></tr>
<tr><td>(B)</td><td>• Firstly, since product moment correlation coefficient = 0.990 is very close to 1, there is a strong positive linear correlation between x^2 and t.
• Secondly, $x = 11$ is within the data range of $2.1 \le x \le 15$, interpolation is done to obtain the estimate value of t. Hence, the estimate value of t is reliable.</td><td>[2]</td></tr>
</table>

<table>
<tr><td>9
(a)</td><td>• The probability of a refrigerator being faulty is constant.
• The event that a refrigerator is faulty is independent of any other refrigerators being faulty.

<u>Comments</u>
A common mistake is to say that the probability that a refrigerator is faulty is independent of the probability that any other refrigerators is faulty.</td><td>[2]</td></tr>
</table>

(b)	$X \sim B(90, 0.02)$ $P(X > 1) = 1 - P(X \le 1)$ $= 1 - 0.46043$ $= 0.53957$ $= 0.540 (3s.f.)$ Comments Key into GC binomcdf(90, 0.02, 1).	[2]
(c)	Let W be the number of days out of 5 days in a working week that Alan finds more than one faulty refrigerator. $W \sim B(5, 0.53957)$ $P(W < 2) = P(W \le 1)$ $= 0.14194$ $= 0.142 (3s.f.)$ Comments Students should avoid using the variable Y as it is used in (e) of this question. Key into GC binomcdf(5, 0.53957, 1).	[2]
(d)	Let V be the number of refrigerators out of 450 that are faulty. $V \sim B(450, 0.02)$ $P(V < 10) = P(V \le 9)$ $= 0.58741$ $= 0.587 (3s.f.)$ Comments Key into GC binomcdf(450, 0.02, 9).	[2]
(e)	$X \sim B(90, 0.02)$ and $Y \sim B(60, 0.03)$ P(exactly two items are faulty) $= P((X = 2) \cap (Y = 0)) + P((X = 1) \cap (Y = 1)) + P((X = 0) \cap (Y = 2))$ Since X and Y are independent, $P((X = 2) \cap (Y = 0)) + P((X = 1) \cap (Y = 1)) + P((X = 0) \cap (Y = 2))$ $= P(X = 2)P(Y = 0) + P(X = 1)P(Y = 1) + P(X = 0)P(Y = 2)$ $= (0.27074)(0.16080) + (0.29812)(0.29840) + (0.16231)(0.27225)$ $= 0.043537 + 0.088961 + 0.044190$ $= 0.17668$ $= 0.177 (3s.f.)$ Comments Use binompdf function in the GC. Students need to take into account the 3 cases which can give rise to exactly two items being faulty.	[3]

	It is advisable that students should show more working steps so that in the event that final answer is incorrect, working marks can be obtained.	

10 (a)	Since the two machines are different, the mass of the packets of flour produced by machine P does not affect the mass of the packets of flour produced by machine Q. Hence, the two distributions are independent.	[1]
(b)	Let A be the mass of a packet of flour produced by machine P in kg. Let C be the mass of a packet of flour produced by machine Q in kg. $A \sim N\left(2.2, 0.1^2\right)$ and $C \sim N\left(2.1, 0.05^2\right)$ $P(A > C) = P(A - C > 0)$ Let Y = A – C $E(Y) = 2.2 - 2.1 = 0.1$ $Var(Y) = 0.1^2 + 0.05^2 = 0.0125$ $Y \sim N(0.1, 0.0125)$ $P(Y > 0) = 0.81445$ $= 0.814(3s.f.)$ Comments Students should avoid using the variable X as it is used in (d). Key into GC normalcdf(0, 1E99, 0.1, square root(0.0125))	[3]
(c)	Let $W = A_1 + A_2 + A_3 + C_1 + C_2 + C_3 + C_4 + C_5$ $E(W) = 3(2.2) + 5(2.1)$ 17.1 $Var(W) = 3\left(0.1^2\right) + 5\left(0.05^2\right)$ $= 0.0425$ $W \sim N(17.1, 0.0425)$ $P(W > 17) = 0.68618$ $= 0.686(3s.f.)$ Comments Key into GC normalcdf(17, 1E99, 17.1, square root(0.0425))	[3]
(d)	Let μ be the population mean mass of a packet of flour produced by machine P after adjustment in kg. H_0 is null hypothesis and H_1 is alternative hypothesis. $H_0 : \mu = 2.2$ against $H_1 : \mu \neq 2.2$ Comments Students should recognise that this is a 2 sided test based on the phrase ‘test if the mean mass of packets produced by that machine now differs from 2.2kg’.	[2]

(e) [2]

Unbiased estimate of population mean

$= 2 + \frac{4.5}{30}$

$= 2.15$

Unbiased estimate of population variance

$= \frac{1}{29}\left(1.11 - \frac{4.5^2}{30}\right)$

$= 0.015$

<u>Comments</u>

I usually recommend students to write out in words and avoid using the symbols such as $\bar{x}$ and σ^2 as these represent sample mean and actual population variance instead. Hence, using the symbol such as $\bar{x}$ is not answering the question directly even though the sample mean value is the same as the unbiased estimate of population mean.

(f) [4]

$H_0 : \mu = 2.2$ against $H_1 : \mu \neq 2.2$

Since sample size n = 30 is large, by Central Limit Theorem,

$\bar{X} \sim N\left(2.2, \frac{0.015}{30}\right)$ and $Z = \frac{\bar{X} - 2.2}{\sqrt{\frac{0.015}{30}}} \sim N(0,1)$ approximately

p value method	Z value method
Reject H_0 when p value ≤ 0.05 p value $= 2P\left(\bar{X} < 2.15\right)$ $= 2(0.012673)$ $= 0.025346$ $= 0.0253 (3s.f.)$ Since p value = 0.0253 < level of significance = 0.05, there is sufficient evidence to reject H_0 and sufficient evidence at the 5% level of significance to conclude that the population mean mass of packets produced by machine P after the adjustment differs from 2.2 kg.	Reject H_0 when $Z_{calc} \leq$ critical Z value = -1.95996 $Z_{calc} = \frac{2.15 - 2.2}{\sqrt{\frac{0.015}{30}}} = -2.2361$ Since Z_{calc} = -2.24 < critical Z value = -1.96, there is sufficient evidence to reject H_0 and sufficient evidence at the 5% level of significance to conclude that the population mean mass of packets produced by machine P after the adjustment differs from 2.2 kg.

<u>Comments</u>

- Central Limit theorem has to be applied here as we cannot simply assume that the mass of the packets of flour produced by machine P continue to follow a normal distribution after the adjustment.
- For Z value method, critical Z value is obtained by keying into GC invNorm(0.025, 0, 1, left)

(g) [1]

After the adjustment, the mass of packets of flour produced by machine P may not necessarily follow a normal distribution anymore. By using a sample of fewer than 30 packets, central limit theorem cannot be applied to approximate the sample mean mass of packets of flour produced by machine P distribution as a normal distribution.

(h) [1]

If the sample is not chosen randomly, each packet of flour produced will not have equal chance of being selected. This can cause the hypothesis test to become biased where the production manager may intentionally select the packets of flour with mass that differs largely from 2.2kg to form the sample.
Hence, this can lead to an erroneous conclusion that the population mean mass of packets of flour produced by machine P after adjustment differs from 2.2kg when it is actually not the case.

END