2017 SINGAPORE-CAMBRIDGE A LEVEL H2 MATH P2 SUGGESTED ANSWER KEY (9758)

Written and Prepared by Mr Mitch Peh

Preface

Dear JC students in Singapore,
Hope you will find this A Level examination solution set useful for your revision.

The answers and comments to this solution set are personally crafted and written by Mr Mitch Peh, an experienced former MOE JC lecturer and tutor in Singapore. Currently, Mr Peh is a full time A Level private tutor, specialising in the teaching of A Level subjects: Physics, Chemistry, Mathematics and Economics at both H1 and H2 Levels. You can find the A Level solutions for the other subjects under the various subject tabs at www.jcpcme.com.

Mr Peh has a proven track record in helping his students achieve success for the A Levels and internal school examinations including promos, advancement tests to JC2, block tests, mid years and prelims. Most of Mr Peh's students achieve "A's and 'B's grades for the A Level examinations. During his stint teaching at St Andrew's Junior College, Mr Peh has helped his classes achieve 100% promotion to JC2 on multiple occasions, attain close to 100% "A"s for H1 Project Work, clinch accolades like "Most Improved Class Award" and "Best Performing Class of the Cohort" for many of the internal school examinations. Mr Peh also has former students who subsequently went on to pursue H3 subjects and enroll in prestigious university courses like Dentistry, Medicine and Law.

If you are interested to be coached by Mr Peh for your preparations towards the A Levels, these are 3 more reasons why you should join Mr Peh's classes:

1. Lessons can be fully customised to your needs
- You have the full autonomy to decide the subject(s), content and pace that you want to cover for each lesson, out of any of the 4 subjects: Physics, Chemistry, Mathematics or Economics.
- Mr Peh will help to analyse your weaknesses in each individual subject and provide personalised feedback and suggestions for improvement.

2. **Answers to your questions can be addressed outside of the classroom**
- If you face any difficulty or challenge doing any of your tutorial questions, simply take a screenshot with your phone and send it to Mr Peh via Whatsapp. Mr Peh will answer your questions in the earliest possible time when he is available.

3. **You only pay the price of 1 subject but enjoy premium coverage for all 4 subjects**.
- Mr Peh provides resources for all 4 subjects including summarised notes, compiled topical questions sourced from past year school prelim examinations, Practical guides for Chemistry & Physics, examination checklists, mock papers etc.
- This is probably the only tuition service in Singapore which allows you to enjoy such extensive coverage and benefits.

Note that Mr Peh only takes in a limited number of students each year. You can find his lesson slots available under "Tuition Services" tab at www.jcpcme.com. For any further enquiries, you can directly whatsapp him at 9651 7737.

For the solution set below, if you find any discrepancies or you have any feedback or comments, please kindly direct them to Mr Peh through Whatsapp at 9651 7737.

The question paper has been omitted due to copyright reasons.

Overall Remarks for 2017 A Level H2 Math Paper 2

- This is another manageable paper where you should be able to do well with strong content knowledge. Drawing diagrams to illustrate will greatly aid you in answering question 6 and 9 even though they are not compulsory.
- For the section on Pure Math, the mathematical manipulations can be a little tedious here especially for questions 1 and 4. My personal advice for students who are easily intimidated by tedious manipulations is to start working on the section on Statistics first.
- Also, most of the Pure Math topics which have not been tested extensively in Paper 1 were tested here: Parametric Equations, Arithmetic Progression and Geometric Progression, Functions, Finding area and volume with integration. Hence, you should be more or less prepared for the topics already.
- There is 1 exception. Even though the topic of graphical transformation has already been tested in Paper 1, it is tested here again in question 3.
- For question 1, you will need to recall your O Level knowledge on the length formula learnt under topic of coordinate geometry and be comfortable working with parametric equations.
- For question 2, you must be able to recall the various formulae learnt under the topic of APGP to answer the questions. You should also know how to solve polynomial equations with the use of the graph method and table table in GC.
- For question 3, strong knowledge in the topics of graphing techniques and transformation, functions will allow you to do well. You should know how to identify self-inverse functions.
- For question 4, you should be familiar with using integration techniques to find area under graph and volume generated by rotating graph about y axis. Manipulations can be tedious here so just be careful.
- For the section on Statistics, there is a strong emphasis on the topic of probability in this paper, tested in questions 5, 6 and 9.
- Other than question 6 on permutation and combination, most of the other Statistics questions are very standard questions where you strive to obtain most of the marks.
- For question 5, there is a mixture of topics being tested here: Probability, finding expected value and variance, binomial distribution. With strong content knowledge, this should be a very manageable question for you.
- For question 6, (ii) can be a challenge to many students as you will need to interpret the information provided by the question correctly and then consider the possible permutations.
- For question 7, this is a standard hypothesis testing question on the use of 2 tailed test. Besides that, you should also be familiar with finding unbiased estimate of population mean and variance value.
- For question 8, choosing the correct model for (b)(i) is important as it will affect your answer for the later parts of the question.
- For question 9, drawing a probability tree will be very helpful to answer the later parts of the question. For the earlier parts of the question, you will need to interpret the information correctly and form the correct binomial distributions and probability inequalities.
- Question 10 is a standard normal distribution question. Again, just make sure that you interpret the information correctly and form the correct normal distributions and probability inequalities.

2017 A Level 9758 H2 Math P2 Suggested Solution

Section A: Pure Mathematics [40 marks]

<table>
<tr><td>1</td><td>(i)</td><td>Topic: Solving parametric equations
Substitute the equation of C in $y = 2x$
$2t = \frac{6}{t} \Rightarrow 2t^2 - 6 = 0$
$\left(t-\sqrt{3}\right)\left(t+\sqrt{3}\right)=0$
Hence, $t = -\sqrt{3}$ or t=$\sqrt{3}$
When $t = -\sqrt{3}$, $x = -\frac{3}{\sqrt{3}}, y = -2\sqrt{3}$
When $t = \sqrt{3}$, $x = \frac{3}{\sqrt{3}}, y = 2\sqrt{3}$
Length AB= $\sqrt{\left(\frac{3}{\sqrt{3}} - \left(-\frac{3}{\sqrt{3}}\right)\right)^2 + \left(2\sqrt{3} - \left(-2\sqrt{3}\right)\right)^2}$
$= \sqrt{\left(\frac{6}{\sqrt{3}}\right)^2 + \left(4\sqrt{3}\right)^2}$
$= \sqrt{12+48}$
$= \sqrt{60}$
$= 2\sqrt{15}$ units
<u>Comments</u>
• We should substitute the parametric equations into the Cartesian equation to solve more efficiently, rather than the other way round.
• Just be careful with the manipulation.
• We need to recall the length formula learnt in the topic of coordinate geometry $\sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}$ in order to find length AB.
• For this part of the question, there is no need to find $\frac{dy}{dx}$ yet as we are only finding the point of intersection.</td><td>[3]</td></tr>
<tr><td></td><td>(ii)</td><td>Topic: Differentiation involving parametric equations
$x = \frac{3}{t}, \quad y=2t \Rightarrow \frac{dx}{dt} = -\frac{3}{t^2}$ and $\frac{dy}{dt} = 2$
Hence, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2 \times \left(-\frac{t^2}{3}\right) = \frac{-2t^2}{3}$</td><td>[5]</td></tr>
</table>

From observation, at the point $\left(\frac{3}{p}, 2p\right)$, t = p

Hence, the equation of tangent: $y-2p=-\frac{2p^2}{3}\left(x-\frac{3}{p}\right)$

To find point D (x intercept), subst y=0

$-2p=-\frac{2p^2}{3}\left(x-\frac{3}{p}\right)$

$\frac{3}{p}=x-\frac{3}{p}\Rightarrow x=\frac{6}{p}$

Hence, the coordinate of point D is $\left(\frac{6}{p}, 0\right)$

To find point E(y intercept), subst x=0

$y-2p=-\frac{2p^2}{3}\left(-\frac{3}{p}\right)=2p$

$y=4p$

Hence, the coordinate of point E is $(0, 4p)$

Given that F is the midpoint of DE, the coordinate of point F is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{3}{p}, 2p\right)$

Hence, the parametric equations traced by F are:

$x=\frac{3}{p}....(1), y=2p...(2)$

From (1), $p=\frac{3}{x}.....(3)$

Subst (3) in (2),

$y=\frac{6}{x}$

Thus the Cartesian equation of the curve traced by F as p varies is $y=\frac{6}{x}$

Comments

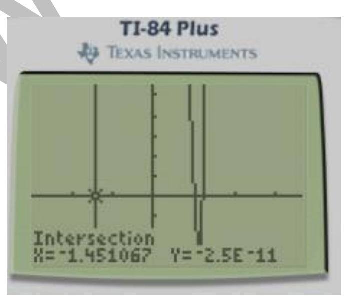
- In this question, you will need to be familiar with the writing of the equation of tangent to curve:
  $y-y_1=m(x-x_1)$
- You will also need to be familiar with Secondary school concept of finding the coordinates of the midpoint between 2 points.
- Another skill required here is to be able to form a cartesian equation from a pair of parametric equations.

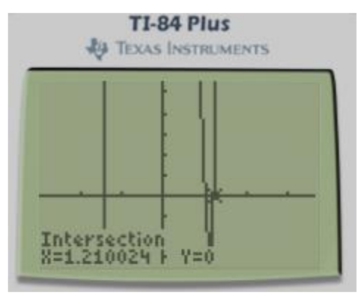
<table>
<tr><td>2</td><td>(i)</td><td>Topic: Arithmetic Progression
Let the common difference be d.
For AP, $S_{13}=\frac{13}{2}\left[2(3)+(12)d\right]=156$
12d=18
d=1.5

<u>Comments</u>
• Simple recall of the summation of AP formula</td><td>[2]</td></tr>
<tr><td></td><td>(ii)</td><td>Topic: Geometric Progression
For GP, $S_{13}=3\frac{\left(r^{13}-1\right)}{r-1}=156$
Dividing by 3 throughout and cross multiply r-1,
$r^{13}-1=52(r-1)$
$r^{13}-52r+51=0(shown)$
r=1 is a solution to the above equation.
However, when common ratio, r = 1, $S_{13}=3\times13=39\neq156$
Hence, r=1 cannot be the common ratio.

$r^{13}-52r+51=0$
From GC, through graph sketching,
r = -1.45 or 1.21

<u>Comments</u>
• Note that we should not say that when r=1, S_{13} is undefined based on the summation of GP formula $S_{13}=3\frac{\left(r^{13}-1\right)}{r-1}$ since the denominator would become 0. Instead, when r=1, the geometric progression goes like this: 1, 1, 1, 1,…., 1,1 for 13 times. Hence, S_{13} should be 13 instead.
• To use the graph sketching function on GC to solve the polynomial equation, we should use Zoombox to see where the graph exactly intersects the *x* axis.




</td><td>[4]</td></tr>
</table>

(iii) Topic: Arithmetic progression and geometric progression [3]

Comparing both the nth term of both the geometric progression and arithmetic progression,

$$ar^{n-1} > 100\left[a+(n-1)d\right]$$

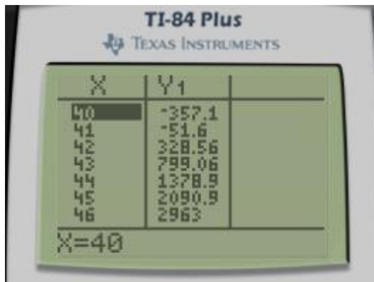
$$3r^{n-1} > 100\left[3+(n-1)(1.5)\right]$$

$$3r^{n-1} > 300+150n-150$$

$$3r^{n-1} - 150n - 150 > 0$$

$$1.21^{n-1} - 50n - 50 > 0$$

From GC table, the smallest possible value of n is 42.



Comments

- We will need to be familiar with the use of the GC to solve the inequality involving the polynomial expression as it cannot be solved manually.

3 (a) Topic: Functions and graphical transformations

(i) To obtain $y = f(2x)$, we replace x with $2x$.
Hence, we scale parallel to the x axis by factor 0.5.
Hence, the coordinates where curves cut the axes are: $\left(\frac{a}{2}, 0\right)$ and $(0, b)$

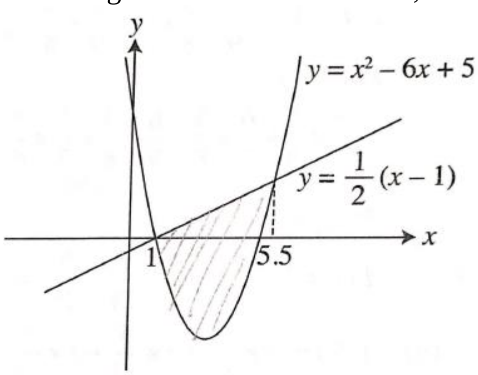
(ii) To obtain $y = f(x-1)$, we replace x with $x-1$.
Hence, we translate 1 unit in the positive x direction.
Hence, the coordinates where the curve cuts the axis is: $(a+1, 0)$

(iii) To obtain $y = f(2x-1)$, we first replace x with $x-1$.
Hence, we translate 1 unit in the positive x direction.
Then we replace x with $2x$
Hence, we scale parallel to the x axis by factor 0.5.
Hence, the coordinates where the curve cuts the axis is: $\left(\frac{a+1}{2}, 0\right)$

(iv) To obtain $y = f^{-1}(x)$, we reflect the graph about the line $y=x$.
Hence, the coordinates where curves cut the axes are: $(0, a)$ and $(\text{b}, 0)$

		Comments • In this question, we will have to be familiar with both topics of graphical transformations and functions in order to know the replacements to perform in the equations and the corresponding actions to take. • For (iv), we should be careful and not mistake the inverse function as reciprocal function $y=\frac{1}{f(x)}$.	
	(b) (i)	State the value of *a* and explain why this value has to be excluded from the domain of g. Topic: Graphing Techniques a=1 as g(1) is undefined. This is because when *x*=1, it will cause the denominator of $\frac{1}{1-x}$ to become 0. Comments • Simple observation on when the function of g will be undefined.	[2]
	(ii)	Topic: Functions $g(x)=1-\frac{1}{1-x}$ $g^2(x)=1-\frac{1}{1-\left(1-\frac{1}{1-x}\right)}=1-(1-x)=x$ Hence, $g^2(x)=x$ where $x\in\mathbb{R}, x\neq a$ since $D_{g^2}=D_g$ Since $g^2(x)=x$ and we also know that $gg^{-1}(x)=x$, this implies that g must be a self-inverse function where $g(x)=g^{-1}(x)$ Hence, $g^{-1}(x)=1-\frac{1}{1-x}$ where $x\in\mathbb{R}, x\neq a$ since $D_{g^{-1}}=R_g$ Comments • To find $g^2(x)$, we must be careful with our substitution in the denominator to avoid careless mistakes. • We will need to be familiar with the concept of self-inverse functions here which is frequently tested at the A Levels. • For self-inverse functions, the domain and the range of the function are the same.	[4]

	(iii)	Topic: Functions $g^2(b) = g^{-1}(b) \Rightarrow \text{b} = 1 - \dfrac{1}{1-b}$ $b = \dfrac{-b}{1-b}$ $b = \dfrac{b}{b-1}$ $b^2 - b - b = 0$ $b(b-2) = 0$ Hence, the values of b are 0 and 2. <u>Comments</u> • Easy question which involves simple substitution of *x* as b and solving the equation.	[2]

4	(a)	Topic: Integration to find area between 2 curves Sketching the two cures on the GC, we have:  The coordinates of the point of intersection is x=1 and x=5.5 Area= $\int_1^{5.5} \dfrac{x-1}{2} - \left(x^2 - 6x + 5\right) dx$ $= \int_1^{5.5} -x^2 + 6.5x - 5.5 dx$ $= \left[-\dfrac{x^3}{3} + \dfrac{6.5x^2}{2} - 5.5x \right]_1^{5.5}$ $= 15.1875 \ units^2$ <u>Comments</u> • We can just use the integration function in GC to solve directly as well since the question did not state that we have to find the exact area of the plate. • There is no need to find the *x* intercepts in this question. We only have to find the point of intersections between the two curves. \	[4]

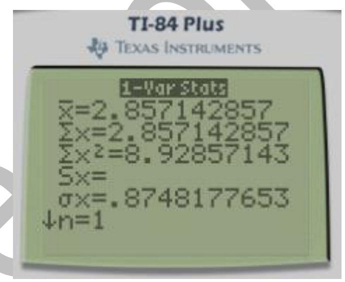
	(b) (i)	Topic: Integration to find volume Volume $= \pi\int_0^1 \left(\frac{\sqrt{y}}{a-y^2}\right)^2 dy = \pi\int_0^1 \frac{y}{\left(a-y^2\right)^2} dy$ $= -\frac{1}{2}\pi\int_0^1 \frac{-2y}{\left(a-y^2\right)^2} dy = -\frac{1}{2}\pi\left[\frac{\left(a-y^2\right)^{-1}}{-1}\right]_0^1$ $= \frac{1}{2}\pi\left[\frac{1}{a-y^2}\right]_0^1$ $= \frac{1}{2}\pi\left(\frac{1}{a-1}-\frac{1}{a}\right)$ $= \frac{1}{2}\pi\left(\frac{a-(a-1)}{a(a-1)}\right) = \frac{\pi}{2a(a-1)}\ units^3$ <u>Comments</u> • There is no need to make *y* the subject here in order to view the shape of the curve on our GC. Instead, we can just base on our knowledge of how integration should be performed to obtain the volume generated by rotating curve about the *y* axis. • We have to form the correct expression to perform the integration, remembering to multiply by π and squaring the function. • We should not expand the denominator within the integral function. Instead, we use the property of $\int f'(x)\left[f(x)\right]^n dx$ to integrate the function. • We will need the correct expression for the volume here in order to obtain the correct answer in (ii).	[4]
	(ii)	Topic: Forming and solving equations The volume generated by this other curved container is $\frac{\pi}{2b\left(b-1\right)}$ units^3. Hence, $\frac{4\pi}{2a\left(a-1\right)} = \frac{\pi}{2b\left(b-1\right)}$ $\frac{2}{a\left(a-1\right)} = \frac{1}{2b\left(b-1\right)}$ Multiplying by $(a^2-a)(2b^2-2b)$ throughout, $4b^2-4b = a^2-a$ $4b^2-4b-a^2+a=0$	[3]

		$b=\dfrac{4\pm\sqrt{(-4)^2-4(4)(-a^2+a)}}{2(4)}$ $b=\dfrac{4\pm\sqrt{16-16(a-a^2)}}{8}=\dfrac{4\pm4\sqrt{1-a+a^2}}{8}$ $=\dfrac{1}{2}+\dfrac{1}{2}\sqrt{1-a+a^2}$ or $\dfrac{1}{2}-\dfrac{1}{2}\sqrt{1-a+a^2}$ Since a>1 and b>1, $\dfrac{1}{2}-\dfrac{1}{2}\sqrt{1-a+a^2}$ is rejected as it will cause the value of b to be smaller than 1. Hence, $b=\dfrac{1}{2}+\dfrac{1}{2}\sqrt{1-a+a^2}$. Comments • As the integration function is the same, with the same limits, we can directly form an equation relating the answer in (i) and the expression for the volume here, instead of performing integration again. • Since the same model is used, if a>1, b will also be >1.	

Section B: Probability and Statistics [60 marks]

5	(i)	Topic: Probability $P(T=2)=\dfrac{6}{9}\times\dfrac{5}{8}=\dfrac{5}{12}$ $P(T=3)=\dfrac{3}{9}\times\dfrac{6}{8}\times\dfrac{5}{7}\times2=\dfrac{5}{14}$ $P(T=4)=\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{6}{7}\times\dfrac{5}{6}\times\dfrac{3!}{2!}=\dfrac{5}{28}$ $P(T=5)=\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\times\dfrac{6}{6}\times\dfrac{5}{5}\times\dfrac{4!}{3!}=\dfrac{1}{21}$ Comments • Note that the counters are taken without replacement here. • The range of T values for 2 red counters to be chosen is from 2 to 5. T=2 occurs when Lee draws 2 red counters consecutively. T=5 occurs when Lee draws 3 yellow counters consecutively before drawing 2 red counters. • P(T=2) occurs when RR is chosen • P(T=3) occurs when TRR is chosen with permutations possible. • P(T=4) occurs when TTRR is chosen with permutations possible. • P(T=5) occurs when TTTRR is chosen with permutations possible. • We can double check our answer we adding all the probability which should sum up to 1.	[3]

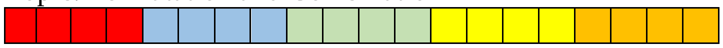
<table>
<tr><td></td><td>(ii)</td><td>Topic: Discrete random variable on finding expected value and variance
$E(T) = 2\times\frac{5}{12} + 3\times\frac{5}{14} + 4\times\frac{5}{28} + 5\times\frac{1}{21} = 2\frac{6}{7}$
$\text{Var}(T) = E\left(T^2\right) - \left(E(T)\right)^2 = 2^2\times\frac{5}{12} + 3^2\times\frac{5}{14} + 4^2\times\frac{5}{28} + 5^2\times\frac{1}{21} - \left(\frac{20}{7}\right)^2$
$= \frac{75}{98}$
<u>Comments</u>
• We can check our answer with the GC, by keying T value in L1, probability values in L2. Then use 1-VarStat, setting list as L1 and frequency list as L2. Note that we need to square the standard deviation value to obtain the variance value.

</td><td>[2]</td></tr>
<tr><td></td><td>(iii)</td><td>Topic: Binomial Distribution
$P(T \geq 4) = P(T = 4) + P(T = 5) = \frac{5}{28} + \frac{1}{21} = \frac{19}{84}$
Let X be the number of games out of 15 where Lee has to take at least 4 counters out of the bag.
$X \sim B\left(15, \frac{19}{84}\right)$
$P(X \geq 5) = 1 - P(X \leq 4) = 0.238(3s.f)$
<u>Comments</u>
• We should observe that we need to apply binomial distribution here and not normal distribution.
• For the calculation of the probability, we key into the GC 1-binomcdf(15, 19/84, 4).</td><td>[2]</td></tr>
</table>

6 (i) [2]

Topic: Permutation and Combination



We group the 4 cards in each family into 1 block. In total, there are 5 blocks.

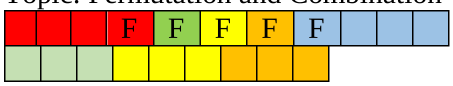
No. of ways = $5!\times(4!)^5 = 955\ 514\ 880$ ways

Comments

- You need to be familiar with the block technique here.

(ii) [3]

Topic: Permutation and Combination



No. of ways = $(3!)^2\times 3!\times 2\times 10! = 1\ 567\ 641\ 600\,ways$

Comments

- Drawing a diagram to illustrate will certainly help here.
- We multiply by 3! twice to arrange the members of Red family and Blue family.
- Then we multiply by 3! one more time to arrange the green, yellow and orange father.
- We multiply by 2 as the Red family can be on the right while the Blue family is on the left instead.
- Lastly, we multiply by 10! to arrange the remaining 9 individuals and the entire block as a whole. A common mistake made by students is to only multiply by 9!, forgetting to take the entire block into account.

(iii) [4]

Topic: Permutation and Combination, Probability

We will slot in the 5 fathers in between the remaining 15 individuals.

Probability= $\dfrac{(15-1)!\times {}^{15}C_5\times 5!}{(20-1)!} = \dfrac{1001}{3876}$

Comments

- There are (15-1)! Ways to arrange the remaining 15 individuals in a circle.
- We multiply by ${}^{15}C_5$ because there are 15 available slots to choose from to slot in the 5 fathers.
- Then we arrange the 5 fathers so we multiply by 5!

7 (i) [1]

Topic: Sampling

- Each biscuit bar has equal probability of being chosen as part of the sample.
- Each selection of the biscuit bar is made independently of the selection of other biscuit bars.

Comments

- This is a descriptive question testing the understanding of the term: random sample
- A common mistake is to mix the 2 statements together and say that the probability of selecting a biscuit bar is independent of the other probability. It should be the event of selecting a biscuit bar is independent of the event of selecting other biscuit bars.

(ii) [2]

Topic: Sampling on calculating unbiased estimate of population mean and variance

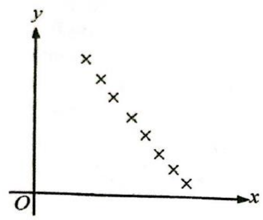
$\bar{x} = 32 - \dfrac{7.7}{40} = 31.8075 = 31.8(3s.f.)$

		$s^2=\frac{1}{n-1}\left\{\sum(x-3.2)^2-\frac{\left[\sum(x-3.2)\right]^2}{n}\right\}$ $=\frac{1}{39}\left[11.05-\frac{(-7.7)^2}{40}\right]$ $=0.24532=0.245(3s.f.)$ <u>Comments</u> • The formula for calculating the unbiased estimate of population variance can be found in MF26.	
	(iii)	Topic: Hypothesis Testing Let X be the mass of a randomly chosen biscuit bar and μ be the population mean mass of biscuit bars in grams. To test H_0: μ =32 against H_1: μ ≠ 32 Use a two tailed z test at 1% level of significance Under H_0, as n=40 is large, by Central Limit Theorem, $\overline{X} \sim N\left(32, \frac{0.24532}{40}\right)$ approximately $Z=\frac{\overline{X}-32}{\sqrt{0.24532/40}} \sim N(0,1)$ approximately p value = $2P\left(\overline{X}<31.8075\right)=$ 0.013969 = 0.0140 (3s.f.) Since p value > 1%, there is insufficient evidence to reject H_0 and insufficient evidence to suggest that the mean mass of biscuit bars is not 32 grams. <u>Comments</u> • We key into the GC: 2normalcdf(-1E99, 31.8075, 32, sqrt(0.24532/40)) to get the p value. • Alternatively, we can obtain the p value from our GC by selecting stats, Z test and keying in μ =32, σ= sqrt(0.24532), $\overline{x}=31.8075$, n=40, two sided test. • Note that when there is insufficient evidence to reject H_0, we cannot go on to make a strong conclusion that the mean mass is 32 grams indeed.	[5]
	(iv)	Explain why there is no need for the production manager to know anything about the population distribution of the masses of the biscuit bars. Topic: Sampling on the use of Central Limit Theorem • Since the sample size, n=40 is large, by Central Limit theorem, the sample mean mass distribution of biscuit bars follows a normal distribution approximately. • Hence, there is no need for the manager to know anything about the population distribution of the masses. <u>Comments</u> • There is no need to know about the actual population mean mass and population variance once we can apply Central Limit Theorem with the appropriate conditions.	[2]

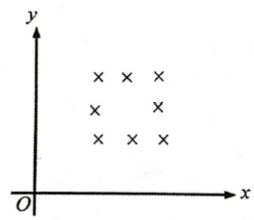
8 (a) [3]

Topic: Correlation and Regression

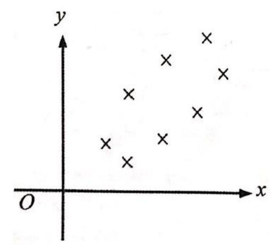
$r = -1$



r=0



r is between 0.5 and 0.9

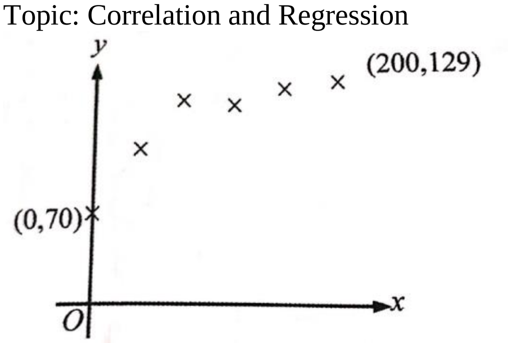


<u>Comments</u>

- For (i), if we draw a downward sloping best fit line, all the points should lie on the best fit line so that the product moment correlation coefficient value is -1.
- For (ii), the points should be lined up in a box shape for product moment correlation coefficient value to be 0.
- For (iii), if we draw an upward sloping best fit line, all the points should lie at some distance away from the best fit line.

(b) (i) [2]

Topic: Correlation and Regression



Hence, the equation of Model D on $y = a\sqrt{x} + b$ provides the most accurate model of the relationship between *x* and *y*.

<u>Comments</u>

- As the value of y is increasing at a decreasing rate, the appropriate models can only be model C or D.
- However, model C: $y = a\ln 2x + b$ requires a vertical asymptote where *x*=0 while model D: $y = a\sqrt{x} + b$ can provide us with a *y* intercept value. Hence, model D is the most accurate model of the relationship.

(ii) [3]

Topic: Correlation and Regression

Using GC,

a= 4.18211 = 4.18 (3s.f.)

b = 74.047 = 74.0 (3s.f.)

r=0.981(3s.f.)

Equation: $y = 4.18\sqrt{x} + 74.0$

<u>Comments</u>

- We need to remember to generate the additional column of $\sqrt{x}$ values in the GC before using the LinReg(a*x*+b) function.

	(iii)	Topic: Correlation and Regression • As x=189 is within the data range of $0 \le x \le 200$, the estimate is obtained by interpolation. Hence, it will give a reliable estimate. • Also, as the product moment correlation coefficient value of 0.981 is close to 1, it suggests a strong positive linear correlation between $\sqrt{x}$ and y. Thus, it will again give a reliable estimate.	[2]

9	(i)	Topic: Binomial Distribution • Each light bulb has the same probability of 0.08 of being faulty • The event that a light bulb is faulty is independent of another light bulb being faulty. Comments • A common mistake made by students is to say that the probability of light bulb being faulty is independent of one another.	[2]
	(ii)	Topic: Binomial Distribution Let X be the number of light bulbs in a box of 12 being faulty. $X \sim B(12, 0.08)$ $P(X \ge 1) = 1 - P(X = 0) = 0.63233 = 0.632 (3s.f.)$ Comments • We need to know how to modify the inequality $P(X \ge 1)$ such that the GC is able to calculate the value for us. • After doing so, we key into the GC: 1-binompdf(12,0.08,0).	[1]
	(iii)	Topic: Probability, Binomial Distribution Method 1: Using Probability Method Probability = $0.63233^{20} = 1.0445 \times 10^{-4} = 1.04 \times 10^{-4} (3s.f.)$ Method 2: Using Binomial Distribution Method Let Y be the number of boxes out of 20 boxes containing at least one faulty light. $Y \sim B(20, 0.63233)$ $P(Y = 20) = 1.04 \times 10^{-4}$ Comments • Method 1 is preferred here as it is less tedious. • For both methods, we should use the more exact value of probability of 0.63233 for the calculations.	[1]
	(iv)	Topic: Binomial Distribution The total number of light bulbs in a carton is 20 x 12 = 240 light bulbs. Let Y be the number of faulty lights in a randomly selected carton of 240 light bulbs. Y~B(240, 0.08) $P(Y \ge 20) = 1 - P(Y \le 19) = 0.45833 = 0.458 (3s.f.)$	[2]

Comments

- Again, we need to know how to modify the expression of $P(Y \geq 20)$ in order to use our GC to calculate the value.
- After doing so, we key into the GC: 1-binomcdf(240, 0.08, 19) to find the value.

(v) [1]

Topic: Probability

- The answer to part (iv) is greater than the answer to part (iii) because the event in (iii) is a subset of the event in (iv).

Comments

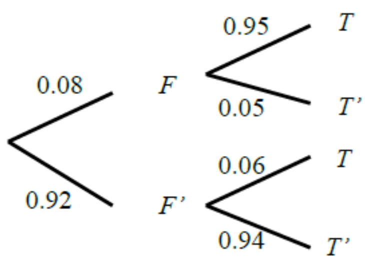
- When each box in one randomly selected carton contains at least one faulty light, there will definitely be at least 20 faulty lights in the carton since there are 20 boxes.
- However, when there is at least 20 faulty lights in the randomly selected carton, it does not necessarily mean that each box in the carton contains at least one faulty light.
- For example, we can just have 2 boxes full of faulty lights while the remaining 18 boxes have no faulty lights at all. This will cause us to have 24 faulty lights altogether, which is greater than the required 20 faulty lights already.

(vi) [3]

Topic: Probability

Let F be the event that the light is faulty

Let T be the event that the light is identified as faulty by the quick test



$$P(F'|\text{T}) = \frac{P(F'\cap T)}{P(T)} = \frac{P(F'\cap T)}{P(\text{F}\cap T)+P(F'\cap T)}$$

$$= \frac{(0.92)(0.06)}{(0.08)(0.95)+(0.92)(0.06)}$$

$$= \frac{69}{164}$$

Comments

- Some students may face difficulty interpreting the question, not knowing that this question actually requires us to find conditional probability value.
- We should construct a probability tree for better clarity in answering the question.
- Based on the information provided in the question, we can fill up all the values in the probability tree.
- Note that the value of 0.95 provided in the question refers to P(T|F) while the value of0.06 refers to P(T|F')
- The final answer can also be written as 0.421(3s.f.)

	(vii)	Topic: Probability $P(F \cap T) + P(F' \cap T') = (0.08)(0.95) + (0.92)(0.94) = 0.941(3s.f.)$ Comments • If we had constructed a probability tree, this question should be easy to answer.	[1]
	(viii)	Topic: Probability Possible Answer 1 • The test is worthwhile as the probability that the quick test correctly identifies the light as faulty or not faulty is very high at 0.941, as obtained in (vii). Possible Answer 2 • The test is not worthwhile as the probability that a light identified as faulty by the quick test but is actually not faulty is significant at 0.421, obtained in (vi). Comments • As long as students make reference to the values obtained in the earlier parts of the question to answer, both answers will be accepted.	[1]

10	(i)	Topic: Normal Distribution Let X be the mass of a randomly selected sphere. X~N(20,0.5^2) $P(X > 20.2) = 0.34457 = 0.345(3s.f.)$ Comments • To find $P(X > 20.2)$, we key into the GC normalcdf(20.2, 1E99, 20, 0.5) • Note that we key in the standard deviation value and not the variance value into our GC.	[1]
	(ii)	Topic: Normal Distribution $E(1.1X) = (1.1)(20) = 22$ $Var(1.1X) = 1.1^2 Var(X) = (1.1)^2 (0.5^2) = 0.3025$ 1.1X~N(22,0.3025) $P(21.5 < 1.1X < 22.45) = 0.61172 = 0.612(3s.f.)$ Comments • We key into the GC: normalcdf(21.5, 22.45, 22, sqrt(0.3025)) • Again, we should remember to key in the standard deviation value and not the variance value into our GC.	[3]
	(iii)	Topic: Normal Distribution Let Y be the mass of a metal bar in grams. Y~N(μ, σ^2) $P(Y > 12.5) = 0.60$ and $P(Y < 12) = 0.25$ $\frac{12.2-\mu}{\sigma} = -0.25334$ and $\frac{12-\mu}{\sigma} = -0.67448$ $12.2 = \mu - 0.25334\sigma$....(1) and $12 = \mu - 0.67448\sigma$.....(2) Using the GC, μ = 12.320 = 12.3 (3s.f.) and σ = 0.47490 = 0.475 (3s.f.)	[4]

		<u>Comments</u> • We need to form simultaneous equations to solve for the value of μ and σ here. • To find the Z values of -0.25334 and -0.67448, we can simply use our GC to calculate. • For older versions of GC, we key in: invNorm(0.40,0,1) and invNorm(0.25,0,1). This is because the GC gives us the Z value that corresponds to the area on the left by default. • For newer versions of GC, we can simply select the region which we are interested in for the invNorm function.	
	(iv)	Topic: Normal Distribution Let $A = 1.1X_1 + 1.1X_2 + Y$ E(A) = 22+22+12.320 = 56.320 Var(A) = $2(0.3025) + 0.47490^2 = 0.83053$ A~N(56.320, 0.83053) P(A > *k*) = 0.75 From GC, *k* = 55.7(3s.f.) <u>Comments</u> • Take note that this question is interested to find the mass of coated sphere so we need to use 1.1X. • The expected value and variance of 1.1X have already been calculated in (ii) of the question. • Potential careless mistake: 0.47490 is the standard deviation value for Y obtained in (iii) of the question and not the variance. Otherwise, we will obtain the incorrect *k* value of 55.6 instead. • Since X and Y are normally distributed, A will also be normally distributed. • The value of *k* can be obtained using GC by keying in invNorm(0.25,56.320, sqrt(0.83053))	[4]

End of Solution